# HW 14.3 #3,10,15,20,23,40,41,45

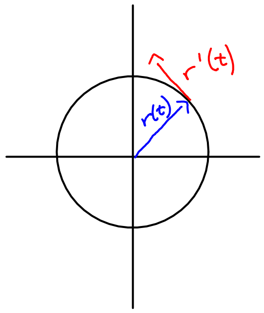
Heather Graehl
MATH 32A Section 1A

Tuesday, July 10, 2007
1:25 PM

**3–8**
(a) Sketch the plane curve with the given vector equation.
(b) Find $\mathbf{r}'(t)$.
(c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of $t$.

**3.** $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $t = \pi/4$

ⓐ




ⓑ $r'(t) = \langle -\sin t, \cos t \rangle$

**9–16** Find the derivative of the vector function.

**10.** $\mathbf{r}(t) = \langle \cos 3t, t, \sin 3t \rangle$

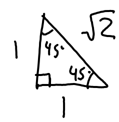
$$r'(t) = \langle -\sin(3t)\cdot 3, 1, \cos(3t)\cdot 3 \rangle$$
$$= \langle -3\sin(3t), 1, 3\cos(3t) \rangle$$

**15.** $\mathbf{r}(t) = \mathbf{a} + t\,\mathbf{b} + t^2\mathbf{c}$

$$r'(t) = b + 2ct$$

**17–20** Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter $t$.

**20.** $\mathbf{r}(t) = 2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \tan t\,\mathbf{k}$, $t = \pi/4$




$$r'(t) = 2\cos t\, i + -2\sin t\, j + \sec^2 t\, k$$
$$r'\left(\tfrac{\pi}{4}\right) = \frac{2}{\sqrt{2}}\, i - \frac{2}{\sqrt{2}}\, j + 2k = \langle \sqrt{2}, -\sqrt{2}, 2 \rangle$$
$$\left|r'\left(\tfrac{\pi}{4}\right)\right| = \sqrt{2+2+4} = \sqrt{8} = 2\sqrt{2}$$
$$T(t) = \frac{\langle \sqrt{2}, -\sqrt{2}, 2 \rangle}{2\sqrt{2}} = \left\langle \frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}} \right\rangle = \boxed{\frac{1}{2} i - \frac{1}{2} j + \frac{1}{\sqrt{2}} k}$$

**23–26** Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

**23.** $x = t^5$, $y = t^4$, $z = t^3$; $(1, 1, 1)$

$$r(t) = \langle t^5, t^4, t^3 \rangle \qquad r(1) =$$
$$r'(t) = \langle 5t^4, 4t^3, 3t^2 \rangle$$
$$\boxed{x = 1 + 5t, \quad y = 1 + 4t, \quad z = 1 + 3t}$$

**40.** Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \sin t\,\mathbf{i} - \cos t\,\mathbf{j} + 2t\,\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

$r(t) = \int r'(t) = \int \langle \sin t, -\cos t, 2t \rangle$

$\int \sin t = -\cos t$

$-\int \cos t = -\sin t$

$\int 2t = \frac{2t^2}{2} = t^2$

$r(0) = -1i + 0j + 0k = i + j + 2k$

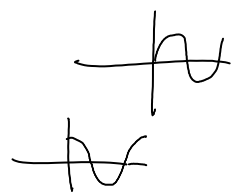
$0 = 2i + j + k$

$$\boxed{r(t) = \langle 2 - \cos t, 1 - \sin t, 2 + t^2 \rangle}$$

41–42 ▪ Find equations of the normal plane and osculating plane of the curve at the given point.

**41.** $x = 2\sin 3t$, $y = t$, $z = 2\cos 3t$; $(0, \pi, -2)$

| $x$ | $y$ | $z$ |
|---|---|---|
| $2\sin 3t = 0$ | $t = \pi$ | $2\cos 3t = -2$ |
| $\sin 3t = 0$ | | $\cos 3t = -1$ |
| $t = \left(\frac{\pi}{2}\right)n$ | | $t = \pi$ |



$r(t) = \langle 2\sin 3t, t, 2\cos 3t \rangle$ $\quad t = \pi$

$r'(t) = \langle 6\cos 3t, 1, -6\sin 3t \rangle$ $\quad t = \pi$

$r'(\pi) = \langle -6, 1, 0 \rangle$

$|r'(\pi)| = \sqrt{37}$

$T(\pi) = \frac{\langle -6, 1, 0 \rangle}{\sqrt{37}}$

$$\boxed{\text{normal plane: } \quad 0 = -6(x - 0) + 1(y - \pi), + 0(z + 2)}$$

$T(t) = \frac{\langle 6\cos 3t, 1, -6\sin 3t \rangle}{\sqrt{37}}$

$T'(t) = \frac{\langle -18\sin 3t, 0, -18\cos 3t \rangle}{\sqrt{37}}$

$|T'(t)| = \frac{\sqrt{18^2\sin^2 3t + 18^2\cos^2 3t}}{\sqrt{37}} = \frac{18\sqrt{\sin^2 3t + \cos^2 3t}}{\sqrt{37}} = \frac{18}{\sqrt{37}}$

$N(t) = \frac{T'(t)}{|T'(t)|} = \frac{\frac{\langle -18\sin 3t, 0, -18\cos 3t \rangle}{\cancel{\sqrt{37}}}}{\frac{18}{\cancel{\sqrt{37}}}} = \langle -\sin 3t, 0, -\cos 3t \rangle$

$N(\pi) = \langle 0, 0, 1 \rangle$

$B(\pi) = \frac{\langle -6, 1, 0 \rangle \times \langle 0, 0, 1 \rangle}{\sqrt{37}}$

$i + 0j + 0k - (+0k + 0i - 6j) = i + 6j$

$$\begin{vmatrix} i & j & k \\ -6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} i & j \\ -6 & 1 \\ 0 & 0 \end{matrix}$$

1 0 0 1 1 0 0

$B(\pi) = \dfrac{\langle 1, 6, 0 \rangle}{\sqrt{37}}$

$\boxed{\text{binormal plane} \quad 1(x-0) + 6(y-\pi) + 0(z-2) = 0}$

**45.** At what point on the curve $x = t^3$, $y = 3t$, $z = t^4$ is the normal plane parallel to the plane $6x + 6y - 8z = 1$?

$r(t) = \langle t^3, 3t, 4t \rangle$ plane $\vec{n} = \langle 6, 6, -8 \rangle$

$T(t) \parallel r'(t)$

$\langle 6, 6, -8 \rangle \parallel \langle 3, 3, -4 \rangle$

$r'(t) = \langle 3t^2, 3, 4 \rangle$

$r(-1) = \boxed{(-1, -3, 1)}$