HW 14.2 #4,14,18,24,29,36,47

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MATH 32A Section 1A

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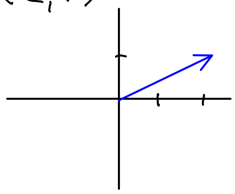
4. $\mathbf{r}(t) = \langle 1 + t, \sqrt{t} \rangle, \quad t = 1$

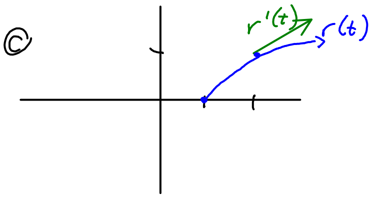
(a) Sketch the plane curve with the given vector equation.
(b) Find $\mathbf{r}'(t)$.
(c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.

(a) $\vec{r}(1) = \langle 2, 1 \rangle$



(b) $\vec{r}\,'(t) = \langle 1, \frac{1}{2\sqrt{t}} \rangle$

(c)



14. $\mathbf{r}(t) = at\cos 3t\,\mathbf{i} + b\sin^3 t\,\mathbf{j} + c\cos^3 t\,\mathbf{k}$ Find the derivative of the function

$$\vec{r}\,'(t) = [a\cos 3t + at(-3\sin 3t)]i + [3b\sin^2 t \cdot \cos t]j + [2c\cos^2 t \cdot -\sin t]k$$

$$\boxed{\langle a\cos 3t - 3at\sin 3t,\ 3b\sin^2 t\cos t,\ -2c\cdot\cos^2 t\sin t \rangle}$$

17–20 Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

18. $\mathbf{r}(t) = 4\sqrt{t}\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}, \quad t = 1$

$$\vec{r}\,'(t) = 4 \cdot \frac{1}{2\sqrt{t}}\,i + 2t\,j + k$$

$$= 2t^{-1/2}\,i + 2t\,j + k$$

$\vec{r}\,'(1) = \langle 2, 2, 1 \rangle$

$|\vec{r}\,'(1)| = \sqrt{4+4+1} = 3$

unit vector $\boxed{\frac{\langle 2, 2, 1 \rangle}{3}}$

23–26 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

24. $x = t^2 - 1, \quad y = t^2 + 1, \quad z = t + 1; \quad (-1, 1, 1)$

$\vec{l}(t) = \langle t^2 - 1, t^2 + 1, t + 1 \rangle$

$\vec{l}\,'(t) = \langle 2t, 2t, 1 \rangle$

$\vec{r} = \langle -1, 1, 1 \rangle + t\langle 2t, 2t, 1 \rangle$ let $t = 0$

$\vec{r} = \langle -1, 1, 1 \rangle + t\langle 0, 0, 1 \rangle$

$= \langle -1, 1, t+1 \rangle$

$\boxed{x = -1 \quad y = 1 \quad z = t + 1}$

29. Determine whether the curve is smooth.

(a) $\mathbf{r}(t) = \langle t^3, t^4, t^5 \rangle$ (b) $\mathbf{r}(t) = \langle t^3 + t, t^4, t^5 \rangle$
(c) $\mathbf{r}(t) = \langle \cos^3 t, \sin^3 t \rangle$

(a) 1. continuous because they are polynomials.

2. $\vec{r}\,'(t) = \langle 3t^2, 4t^3, 5t^4 \rangle$ $\quad \vec{r}\,'(0) = \vec{0}$

$\boxed{\text{not smooth}}$

(b) 1. continuous because they are polynomials

2. $\vec{r}\,'(t) = \langle 3t^2 + 1, 4t^3, 5t^4 \rangle$

$\boxed{\text{yes, smooth}}$

(c) 1. continuous

2. $\vec{r}\,'(t) = \langle 3\cos^2 t \cdot -\sin t, 3\sin^2 t \cdot \cos t \rangle$

$\vec{r}\,'(0) = \vec{0}$

$\boxed{\text{not smooth}}$

36. $\int_1^4 \left(\sqrt{t}\,\mathbf{i} + te^{-t}\,\mathbf{j} + \frac{1}{t^2}\,\mathbf{k} \right) dt$ Evaluate the Integral

i $\int t^{1/2}\,dt = \frac{2t^{3/2}}{3}$

k $\int t^{-2}\,dt = -t^{-1}$

j $\int te^{-t}\,dt$ let $u = t$ $\quad du = dt$

$dv = e^{-t}$ $\quad v = -e^{-t}$

$-te^{-t} - \int -e^{-t}\,dt$

$-te^{-t} + \int e^{-t}\,dt$

$-te^{-t} - e^{-t}$

$$\left\langle \frac{2t^{3/2}}{3}, -te^{-t} - e^{-t}, -t^{-1} \right\rangle \Big|_1^4$$

$$\left\langle \frac{16}{3}, -4e^{-4} - e^{-4}, -\frac{1}{4} \right\rangle - \left\langle \frac{2}{3}, -e^{-1} - e^{-1}, -1 \right\rangle$$

$$\boxed{\left\langle \frac{14}{3}, -5e^{-4} + 2e^{-1}, \frac{3}{4} \right\rangle}$$

47. Show that if $\mathbf{r}$ is a vector function such that $\mathbf{r}''$ exists, then

$$\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)$$

1. by chain rule

$$\frac{d}{dt}[r(t) \times r'(t)] = r(t) \times r''(t) + r'(t) \times r'(t)$$

2. $r'(t) \times r'(t)$ are parallel vectors thus their cross product is $\vec{0}$.

so $\frac{d}{dt}[r(t) \times r'(t)] = r(t) \times r''(t)$