

HW 13.5 #2,10,14,20,34,41,53,63

Monday, July 02, 2007
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MATH 32A Section 1A

2-5 Find a vector equation and parametric equations for the line.

2. The line through the point $(1, 0, -3)$ and parallel to the vector $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

vector equation

$$\vec{r} = \vec{r}_0 + t\vec{v}$$

$$\vec{r}_0 = \langle 1, 0, -3 \rangle$$

$$\vec{v} = \langle 2, -4, 5 \rangle$$

$$\vec{r} = \langle 1, 0, -3 \rangle + t\langle 2, -4, 5 \rangle$$

parametric equation

$$\vec{v} = \langle 2, -4, 5 \rangle \quad \vec{r}_0 = \langle 1, 0, -3 \rangle$$

$$x = 1 + 2t$$

$$y = -4t$$

$$z = -3 + 5t$$

Find parametric and symmetric equations of the line:

10. The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$

$$\langle 1, 1, 0 \rangle \times \langle 0, 1, 1 \rangle = \perp$$

$$\mathbf{i} - 0\mathbf{j} + \mathbf{k} - (0\mathbf{k} + 0\mathbf{i} + \mathbf{j}) = \mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\vec{r}_0 = \langle 2, 1, 0 \rangle \quad \vec{v} = \langle 1, -1, 1 \rangle$$

parametric

$$x = 2 + t$$

$$y = 1 - t$$

$$z = t$$

symmetric

$$t = \frac{x-2}{1} = \frac{-y+1}{1} = z$$

14. Is the line through $(4, 1, -1)$ and $(2, 5, 3)$ perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$?

Let line 1 (\vec{a}) consist of points $(4, 1, -1)$ and $(2, 5, 3)$ and line 2 (\vec{b}) consist of points $(-3, 2, 0)$ and $(5, 1, 4)$.

line 1 $\vec{a} = \langle -2, 4, 4 \rangle$

line 2 $\vec{b} = \langle 8, -1, 4 \rangle$

$$\vec{a} \cdot \vec{b} = 0 \Rightarrow \perp$$

$$\vec{a} \cdot \vec{b} = -16t - 4 + 16 = 4 \neq 0$$

not \perp

19-22 ■ Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

20. $L_1: x = 1 + 2t, y = 3t, z = 2 - t$

$L_2: x = -1 + s, y = 4 + s, z = 1 + 3s$

$$L_1 \left\{ \begin{array}{l} \vec{r}_0 = \langle 1, 0, 2 \rangle \\ \vec{v} = \langle 2, 3, -1 \rangle \end{array} \right.$$

$$L_2 \left\{ \begin{array}{l} \vec{r}_0 = \langle -1, 4, 1 \rangle \\ \vec{v} = \langle 1, 1, 3 \rangle \end{array} \right.$$

$$\vec{L}_1 \cdot \vec{L}_2 = 0 \Rightarrow \perp$$

$$\vec{L}_1 \times \vec{L}_2 = 0 \Rightarrow =$$

if neither satisfied \Rightarrow skew

$$\vec{L}_1 \cdot \vec{L}_2 = 2 + 3 - 3 = 2 \neq 0 \not\Rightarrow \perp$$

$$\vec{L}_1 \times \vec{L}_2 = 9i - j + 2k - (3k - i + 6j) = 10i + 5j - k \neq 0 \not\Rightarrow =$$

$$\begin{array}{ccc|ccc} i & j & k & i & j & k \\ 2 & 3 & -1 & 2 & 3 & -1 \\ 1 & 1 & 3 & 1 & 1 & 3 \end{array}$$

skew

34. The plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t, y = 1 + t, z = 2 - t$

$$t = 0 \Rightarrow (0, 1, 2)$$

$$t = 1 \Rightarrow (3, 2, 1)$$

$$i + 6j + 0k - (3k + 4i + 0j) = -3i + 6j - 3k = \vec{n}$$

$$\begin{array}{ccc|ccc} i & j & k & i & j & k \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 3 & 2 & -1 & 3 & 2 & -1 \end{array}$$

$$\vec{n} \cdot (\vec{r} - \vec{r}_0)$$

$$\langle -3, 6, -3 \rangle \cdot (\vec{r} - \langle 1, 2, 3 \rangle) = 0$$

$$\vec{r} = \langle x, y, z \rangle$$

$$-3x + 6y - 3z - (-3 + 12 - 9) = 0$$

$$-3x + 6y - 3z = 0$$

$$-3x + 6y - 3z = 0$$

Find the point in which the line intersects with the plane.

41. $x = y - 1 = 2z$; $4x - y + 3z = 8$

$$\begin{aligned} x = t & \quad 4t - (1+t) + 3\left(\frac{t}{2}\right) = 8 \\ y = 1+t & \quad 4t - 1 - t + \frac{3}{2}t = 8 \\ z = \frac{t}{2} & \quad 3t + \frac{3}{2}t = 9 \\ & \quad \frac{9}{2}t = 9 \\ & \quad t = 2 \end{aligned}$$

$$x = 2, y = 3, z = 1 \quad (2, 3, 1)$$

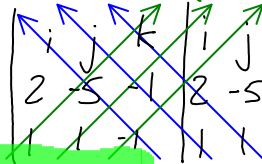
53-54 Find parametric equations for the line of intersection of the planes.

53. $z = x + y$, $2x - 5y - z = 1$

$$\begin{aligned} \text{set } x = 0 \quad z = y & \quad 5y + z = -1 \\ & \quad 6y = -1 \\ & \quad y = -\frac{1}{6} \\ & \quad z = -\frac{1}{6} \end{aligned}$$

$$(x_0, y_0, z_0) = \left(0, -\frac{1}{6}, -\frac{1}{6}\right)$$

$$\langle 2, -5, -1 \rangle \times \langle 1, 1, -1 \rangle = 5i - j + 2k - (-5k - i - 2j) = 6i + j + 7k$$



$$\langle 6, 1, 7 \rangle$$

$$x = 6t \quad y = -\frac{1}{6} + t \quad z = -\frac{1}{6} + 7t$$

63-64 Use the formula in Exercise 39 in Section 12.4 to find the distance from the point to the given line.

63. $(1, 2, 3)$; $x = 2 + t$, $y = 2 - 3t$, $z = 5t$ $Q = (x_0, y_0, z_0) = (2, 2, 0)$

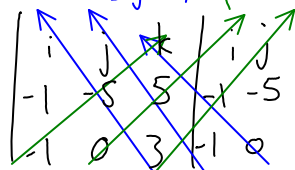
$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

$$R = (1, -3, 5)$$

$$P = (1, 2, 3)$$

$$\mathbf{a} = \vec{QR} = \langle -1, -5, 5 \rangle, \quad \mathbf{b} = \vec{QP} = \langle -1, 0, 3 \rangle$$

$$\mathbf{a} \times \mathbf{b} = -15i - 5j - k - (5k + 5i - 3j) = -20i - 2j - 6k$$



$$|\mathbf{a}| = \sqrt{1 + 25 + 25} = \sqrt{51}$$

$$\langle 20, -2, -6 \rangle$$

$$\frac{\langle 20, -2, -6 \rangle}{\sqrt{51}}$$