HW 13.5 #2,10,14,20,34,41,53,63

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MATH 32A Section 1A

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2–5 ▪ Find a vector equation and parametric equations for the line.

2. The line through the point $(1, 0, -3)$ and parallel to the vector $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

vector equation

$$\vec{r} = \vec{r}_0 + t\vec{v}$$

$$\vec{r}_0 = \langle 1, 0, -3 \rangle$$

$$\vec{v} = \langle 2, -4, 5 \rangle$$

$$\boxed{\vec{r} = \langle 1, 0, -3 \rangle + t\langle 2, -4, 5 \rangle}$$

parametric equation

$$\vec{v} = \langle 2, -4, 5 \rangle \qquad \vec{r}_0 = \langle 1, 0, -3 \rangle$$

$$\boxed{\begin{aligned} x &= 1 + 2t \\ y &= -4t \\ z &= -3 + 5t \end{aligned}}$$

Find parametric and symmetric equations of the line:

10. The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$

$$\langle 1, 1, 0 \rangle \times \langle 0, 1, 1 \rangle = \perp$$

$$i - 0j + k - (0k + 0i + j) = i - j + k$$

$$\left|\begin{array}{ccc} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right| \begin{array}{cc} i & j \\ 1 & 1 \\ 0 & 1 \end{array}$$

$$\vec{r}_0 = \langle 2, 1, 0 \rangle \qquad \vec{v} = \langle 1, -1, 1 \rangle$$

parametric

$$\boxed{\begin{aligned} x &= 2 + t \\ y &= 1 - t \\ z &= t \end{aligned}}$$

Symmetric

$$t = \boxed{\frac{x-2}{1} = \frac{-y+1}{1} = z}$$

14. Is the line through $(4, 1, -1)$ and $(2, 5, 3)$ perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$?

Let line 1 $(\vec{a})$ consist of points (4,1,-1) and (2,5,3) and line 2 $(\vec{b})$ consist of points (-3,2,0) and (5,1,4).

line 1 $\quad \vec{a} = \langle -2, 4, 4 \rangle$

line 2 $\quad \vec{b} = \langle 8, -1, 4 \rangle$

$\vec{a}\cdot\vec{b} = 0 \Rightarrow \perp$

$\vec{a}\cdot\vec{b} = -16+-4+16 = 4 \neq 0$

$\boxed{\text{not } \perp}$

19–22 ▪ Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

20. L_1: $x = 1 + 2t, \quad y = 3t, \quad z = 2 - t$

L_2: $x = -1 + s, \quad y = 4 + s, \quad z = 1 + 3s$

L_1 $\vec{r}_0 = \langle 1,0,2\rangle$

$\vec{v} = \langle 2,3,-1\rangle$

L_2 $\vec{r}_0 = \langle -1,4,1\rangle$

$\vec{v} = \langle 1,1,3\rangle$

$\vec{L}_1\cdot\vec{L}_2 = 0 \Rightarrow \perp$

$\vec{L}_1\times\vec{L}_2 = 0 \Rightarrow =$

if neither satisfied $\Rightarrow$ skew

$\vec{L}_1\cdot\vec{L}_2 = 2+3-3 = 2 \neq 0 \not\Rightarrow \perp$

$\vec{L}_1\times\vec{L}_2$ $9i - j + 2k - (3k - i + 6j) = 10i + 5j - k \neq 0 \not\Rightarrow =$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 2 & 3 & -1 & 2 & 3 \\ 1 & 1 & 3 & 1 & 1 \end{array}\right.$$

$\boxed{\text{skew}}$

34. The plane that passes through the point (1, 2, 3) and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$

$t = 0 \Rightarrow (0,1,2)$

$t = 1 \Rightarrow (3,2,1)$

$i + 6j + 0k - (3k + 4i + 0j) = -3i + 6j - 3k = \vec{n}$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 1 & 2 & 0 & 1 \\ 3 & 2 & 1 & 3 & 2 \end{array}\right.$$

$\vec{n}\cdot(\vec{r}-\vec{r}_0)$

$\langle -3,6,-3\rangle\cdot(\vec{r}-\langle 1,2,3\rangle) = 0$

$\vec{r} = \langle x,y,z\rangle$

$-3x+6y-3z-(-3+12-9) = 0$

$\boxed{-3x+6y-3z = 0}$

$$\boxed{-3x+6y-3z=0}$$

Find the point in which the line intersects with the plane.

41. $x = y - 1 = 2z$; $4x - y + 3z = 8$

$x = t$

$y = 1+t$

$z = \frac{t}{2}$

$$4t-(1+t)+3\left(\tfrac{t}{2}\right)=8$$
$$4t-1-t+\tfrac{3}{2}t=8$$
$$3t+\tfrac{3}{2}t=9$$
$$\tfrac{9}{2}t=9$$
$$t=2$$

$x=2,\ y=3,\ z=1$ $\quad \boxed{(2,3,1)}$

53–54 Find parametric equations for the line of intersection of the planes.

53. $z = x + y$, $2x - 5y - z = 1$

set $x=0$ $\quad z=y$

$$5y+z=-1$$
$$6y=-1$$
$$y=-1/6$$
$$z=-1/6$$

$$(x_0, y_0, z_0) = \left(0, -\tfrac{1}{6}, -\tfrac{1}{6}\right)$$

$$\langle 2,-5,-1\rangle \times \langle 1,1,-1\rangle = 5i-j+2k-(-5k-i-2j) = 6i+j+7k$$

$\langle 6,1,7\rangle$

$$\left|\begin{array}{ccc} i & j & k \\ 2 & -5 & -1 \\ 1 & 1 & -1 \end{array}\right|\begin{array}{cc} i & j \\ 2 & -5 \\ 1 & 1 \end{array}$$

$$\boxed{x=6t \quad y=-\tfrac{1}{6}+t \quad z=-\tfrac{1}{6}+7t}$$

63–64 Use the formula in Exercise 39 in Section 12.4 to find the distance from the point to the given line.

63. $(1, 2, 3)$; $x = 2 + t$, $y = 2 - 3t$, $z = 5t$

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

$Q=(x_0\ y_0\ z_0)=(2,2,0)$

$R = (1,-3,5)$

$P=(1,2,3)$

$a=\overrightarrow{QR}=\langle -1,-5,5\rangle$, $b=\overrightarrow{QP}=\langle -1,0,3\rangle$

$$a\times b = -15i-5j-k-(5k+5i-3j) = -20i-2j-6k$$

$$\left|\begin{array}{ccc} i & j & k \\ -1 & -5 & 5 \\ -1 & 0 & 3 \end{array}\right|\begin{array}{cc} i & j \\ -1 & -5 \\ -1 & 0 \end{array}$$

$$|a| = \sqrt{1+25+25} = \sqrt{51}$$

$$\boxed{\langle 20,-2,-6\rangle}$$

$$\boxed{\frac{\langle 20, -2, -6 \rangle}{\sqrt{51}}}$$